Unit 07

LINEAR EQUATIONS AND INEQUALITIES

Define Linear Equations

A linear equation in one unknown variable x is an equation of the form

$ax + b = 0$, where $a, b \in R$ and $a \neq 0$.

A solution to a linear equation is any replacement or substitution for the variable x that makes the statement true. Two linear equations are said to be equivalent if they have exactly the same solution.

Example

Solve the equation $\frac{3x}{2} - \frac{x-2}{3} = \frac{25}{6}$

Solution

Multiplying each side of the given equation by 6

$$\begin{aligned} 9x-2(x-2) &= 25 \\ \Rightarrow \quad 9x-2x+4 &= 25 \\ \Rightarrow \quad 7x &= 21 \\ \Rightarrow \quad x &= 3 \end{aligned}$$

Check

Substituting x = 3 in original equation,

$$\begin{aligned} \frac{3}{2}(3) - \frac{3-2}{3} &= \frac{25}{6} \\ \frac{9}{2} - \frac{1}{3} &= 25 \\ \frac{25}{6} &= \frac{25}{6} \end{aligned}$$

Since $x = 3$ makes the original statement true, therefore the solution is correct.

Note

Some fractional equations may have no solution.

Example

Solve $\frac{3}{y-1} - 2 = \frac{3y}{y-1}$, $y \neq 1$

Solution

Multiplying both sides by y – 1, we get

$$\begin{aligned} 3-2(y-1) &= 3y \\ \Rightarrow \quad 3-2y+2 &= 3y \\ \Rightarrow \quad -5y &= -5 \\ \Rightarrow \quad y &= 1 \end{aligned}$$

Check

Substituting y = 1 in the given equation, we have

$$\begin{aligned} \frac{3}{1-1} - 2 &= \frac{3(1)}{1-1} \\ \frac{3}{0} - 2 &= \frac{3}{0} \end{aligned}$$

But $\frac{3}{0}$ is undefined. So $y=1$ cannot be a solution.

Thus the given equation has not solution.

Example

Solve $\frac{3x-1}{3} - \frac{2x}{x-1} = x$, $x \neq 1$

Solution

Multiplying each side by $3(x-1)$

$$(x-1)(3x-1)-6x = 3x(x-1)$$
$$\Rightarrow 3x^2-4x+1-6x = 3x^2-3x$$
$$\Rightarrow -10x+1 = -3x$$
$$\Rightarrow -7x = -1$$
$$\Rightarrow x = \frac{1}{7}$$

Check

On substituting $x=\frac{1}{7}$ the original equation is verified a true statement. That means the restriction $x\neq 1$ has no effect on the solution because $\frac{1}{7}\neq 1$.

Hence our solution $x=\frac{1}{7}$ is correct.

Define Radical equation?

When the variable in an equation occurs under a radical, the equation is called a radical equation.

Example

Solve the equations

(a) $\sqrt{2x-3}-7=0$

(b) $\sqrt[3]{3x+5}=\sqrt[3]{3x+5}=\sqrt[3]{x-1}$

Solution

(a) To isolate the radical, we can rewrite the given equation as

$$\sqrt{2x-3} = 7$$
$$\Rightarrow 2x-3 = 49 \quad .....$$
$$\Rightarrow 2x = 52 \Rightarrow x = 26$$

Check

Let us substitute $x=26$ in the original equation. Then

$$\sqrt{2(26)-3}-7 = 0$$
$$\sqrt{52-3}-7 = 0$$
$$\sqrt{49}-7 = 0$$
$$0 = 0$$

Hence the solution set is {26}.

(b) We have

$$\sqrt[3]{3x+5} = \sqrt[3]{x-1}$$

Taking cube of each side

$$\Rightarrow 3x+5 = x-1,$$
$$\Rightarrow 2x=-06 \Rightarrow x=-3$$

Check

We substitute $x = -3$ in the original equation. Then

$$\sqrt[3]{3(-3)+5} = \sqrt[3]{-3-1}$$
$$\sqrt[3]{-9+5} = \sqrt[3]{-4}$$
$$\Rightarrow \sqrt[3]{-4} = \sqrt[3]{-4}$$

Thus $x = -3$ satisfies the original equation.

Here $\sqrt[3]{-4}$ is a real number because we raised each side of the equation to an odd power.

Thus the solution set = {−3}

Example

Solve and check: $\sqrt{5x-7}-\sqrt{x+10}=0$

Solution

When two terms of a radical equation contain variables in the radicand, we express the equation such that one of these terms is on each side. So we rewrite the equation in this form to get

$$\sqrt{5x-7}-\sqrt{x+10}=0$$

Squaring each side

$$5x-7 = x+10,$$
$$5x-x=10+7$$
$$4x=17 \Rightarrow x=\frac{17}{4}$$

Check

Substituting $x=\frac{17}{4}$ in original equation

$$\sqrt{5x-7}-\sqrt{x+10} = 0$$

$$\sqrt{5\left(\frac{17}{4}\right)-7}-\sqrt{\frac{17}{4}+10} = 0$$

$$\sqrt{\frac{57}{4}}-\sqrt{\frac{57}{4}} = 0$$

$$0 = 0$$

i.e., $x=\frac{17}{4}$ makes the given equation a true statement.

Thus solution set $=\left\{\frac{17}{4}\right\}$.

Example

Solve $\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$

Solution

$$\sqrt{x+7}+\sqrt{x+2}=\sqrt{6x+13}$$

Squaring both sides we get

$$x+7+x+2+2\sqrt{(x+7)(x+2)}=6x+13$$

$$\Rightarrow \quad 2\sqrt{x^2+9x+14} = 4x+4$$

$$\Rightarrow \quad \sqrt{x^2+9x+14} = 2x+2$$

Squaring again

$$x^2+9x+14 = 4x^2+8x+4$$

$$\Rightarrow \quad 3x^2-x-10 = 0$$

$$\Rightarrow \quad 3x^2-6x+5x-10 = 0$$

$$\Rightarrow \quad 3x(x-2)+5(x-2) = 0$$

$$\Rightarrow \quad (x-2)(3x+5) = 0$$

$$\Rightarrow \quad x=2, \frac{-5}{3}$$

On checking, we see that $x=2$ satisfies the equation, but $x=\frac{-5}{3}$ does not satisfy the equation. So solution set is $\{2\}$ and $x=\frac{-5}{3}$ is an extraneous root.

Exercise 7.1

Q1. Solve the following equations.

i) $\frac{2}{3}x-\frac{1}{2}x=x+\frac{1}{6}$

Sol: Multiplying both sides by 6

$$^{2}\cancel{6}\left(\frac{2}{\cancel{3}}x\right)-{}^{3}\cancel{6}\left(\frac{1}{\cancel{2}}x\right)=6(x)+\cancel{6}\left(\frac{1}{\cancel{6}}\right)$$

$$4x-3x=6x+1$$

$$x=6x+1$$

$$-1=6x-x$$

$$-1=5x$$

$$\Rightarrow \quad \boxed{x=-\frac{1}{5}}$$

Check:

Substituting $x=-\frac{1}{5}$ in the given equation

$$\frac{2}{3}\left(-\frac{1}{5}\right)-\frac{1}{2}\left(-\frac{1}{5}\right)=-\frac{1}{5}+\frac{1}{6}$$

$$-\frac{2}{15}+\frac{1}{10}=-\frac{1}{5}+\frac{1}{6}$$

$$\frac{-4+3}{30}=\frac{-6+5}{30}$$

$$-\frac{1}{30}=-\frac{1}{30} \text{ which is true}$$

Hence solution set = $\left\{-\frac{1}{5}\right\}$

ii) $$\frac{x-3}{3}-\frac{x-2}{2}=-1$$

Multiplying both sides by 6

$$^{2}\cancel{6}\left(\frac{x-3}{\cancel{3}}\right)-{}^{3}\cancel{6}\left(\frac{x-2}{\cancel{2}}\right)=6(-1)$$

$$2x-\cancel{6}-3x+\cancel{6}=-6$$

$$-x=-6$$

$$\boxed{x=6}$$

Check:

Substituting $x=6$ in the given equation

$$\frac{6-3}{3}-\frac{6-2}{2}=-1$$

$$\frac{3}{3}-\frac{4}{2}=-1$$

$$1-2=-1$$

$-1=-1$ which is true, so solution set = $\{6\}$

iii) $$\frac{1}{2}\left(x-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-3x\right)$$

$$\frac{1}{2}x-\frac{1}{12}+\frac{2}{3}=\frac{5}{6}+\frac{1}{6}-\frac{1}{\cancel{3}}(\cancel{3}x)$$

Multiplying both sides by 12

$$\cancel{12}\left(\frac{1}{\cancel{2}}x\right)-\cancel{12}\left(\frac{1}{\cancel{12}}\right)+{}^{4}\cancel{12}\left(\frac{2}{\cancel{3}}\right)={}^{2}\cancel{12}\left(\frac{5}{\cancel{6}}\right)+{}^{2}\cancel{12}\left(\frac{1}{\cancel{6}}\right)-12(x)$$

$$6x-1+8=10+2-12x$$

$$6x+7=12-12x$$

$$6x+12x=12-7$$

$$18x=5$$

$$\boxed{x=\frac{5}{18}}$$

Check:

Substituting $x=\frac{5}{18}$ in the given equation

$$\frac{1}{2}\left(\frac{5}{18}-\frac{1}{6}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{1}{2}-\cancel{3}\times\frac{5}{{}^{6}\cancel{18}}\right)$$

$$\frac{1}{2}\left(\frac{5-3}{18}\right)+\frac{2}{3}=\frac{5}{6}+\frac{1}{3}\left(\frac{3-5}{6}\right)$$

$$\frac{1}{2}\left(\frac{2}{18}\right)+\frac{2}{3}=\frac{5}{6}-\frac{2}{18}$$

$$\frac{1+12}{18}=\frac{15-2}{18}$$

$$\frac{13}{18}=\frac{13}{18} \text{ which is true, so}$$

Solution set = $\left\{\frac{5}{18}\right\}$

(iv) $$x+\frac{1}{3}=2\left(x-\frac{2}{3}\right)-6x$$

$$x+\frac{1}{3}=2x-\frac{4}{3}-6x$$

Multiplying both sides by 3

$$3x+\cancel{3}\times\frac{1}{\cancel{3}}=3(2x)-\cancel{3}\left(\frac{4}{\cancel{3}}\right)-3(6x)$$

$$3x+1=6x-4-18x$$

$$3x+1=-12x-4$$

$$15x=-5$$

$$x=-\frac{5}{15}$$

$$\boxed{x=-\frac{1}{3}}$$

Check:

Substituting $x=-\frac{1}{3}$ in the given equation

$$-\frac{1}{3}+\frac{1}{3}=2\left(-\frac{1}{3}-\frac{2}{3}\right)-\cancel{6}\left(-\frac{1}{\cancel{3}}\right)$$

$$0=2\left(-\frac{\cancel{3}}{\cancel{3}}\right)+2$$

$0=-2+2$

$0=0$ which is true, so

Solution set $=\left\{-\frac{1}{3}\right\}$

v) $$\frac{5(x-3)}{6}-x=1-\frac{x}{9}$$

Multiplying both sides by 18

$$^{3}\cancel{18}\times\frac{5(x-3)}{\cancel{6}}-18x=18-{}^{2}\cancel{18}\left(\frac{x}{\cancel{9}}\right)$$

$$15(x-3)-18x=18-2x$$

$$15x-45-18x=18-2x$$

$$15x-18x+2x=18+45$$

$$-x=63$$

$$\Rightarrow\ x=-63$$

Check:

Substituting $x=-63$ in the given equation

$$\frac{5(-63-3)}{6}-(-63)=1-\frac{(-63)}{9}$$

$$5\frac{\left(\cancel{-66}^{\,11}\right)}{\cancel{6}}+63=1+\frac{\cancel{63}^{\,7}}{\cancel{9}}$$

$$-55+63=1+7$$

$8=8$ which is true, so

Solution set $=\{-63\}$

vi) $$\frac{x}{3x-6}=2-\frac{2x}{x-2}$$

$$\frac{x}{3(x-2)}=2-\frac{2x}{x-2}$$

Multiplying both sides by $3(x-2)$

$$\cancel{3}(x-2)\times\frac{x}{\cancel{3}(x-2)}=2\times3(x-2)-\frac{2x}{x-2}\times3(x-2)$$

$$x=6x-12-6x$$

$$\boxed{x=-12}$$

Check:

Substituting $x=-12$ in the given equation

$$\frac{-12}{3(-12)-6}=2-\frac{2(-12)}{-12-2}$$

$$\frac{-12}{-36-6}=2-\frac{(\cancel{-24})}{\cancel{-14}}$$

$$\frac{-12}{-42}=2-\frac{12}{7}$$

$$\frac{2}{7}=\frac{14-12}{7}$$

$\frac{2}{7}=\frac{2}{7}$ which is true, so

Solution Set $=\{-12\}$

vii) $$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{4x+10}\quad,\quad x\neq-\frac{5}{2}$$

$$\frac{2x}{2x+5}=\frac{2}{3}-\frac{5}{2(2x+5)}$$

Multiplying both sides by $6(2x+5)$

$$6\cancel{(2x+5)}\times\frac{2x}{\cancel{2x+5}}=\frac{2}{\cancel{3}}\times{}^{2}\cancel{6}\,(2x+5)-\frac{5}{\cancel{2(2x+5)}}\times{}^{3}\cancel{6}\,\cancel{(2x+5)}$$

$$12x=8x+20-15$$

$$12x-8x=5$$

$$4x=5$$

$$4x=5$$

$$x=\frac{5}{4}$$

Check:

Substituting $x=\frac{5}{4}$ in the given equation

$$\frac{\cancel{2}\left(\frac{5}{\cancel{4}}\right)}{\cancel{2}\left(\frac{5}{\cancel{4}}\right)+5}=\frac{2}{3}-\frac{5}{\cancel{4}\left(\frac{5}{\cancel{4}}\right)+10}$$

$$\frac{\frac{5}{\cancel{2}}}{\frac{5+10}{\cancel{2}}}=\frac{2}{3}-\frac{\cancel{5}}{\cancel{15}}$$

$$\frac{\cancel{5}}{\cancel{15}}=\frac{2}{3}-\frac{1}{3}$$

$\frac{1}{3}=\frac{1}{3}$ which is true, so

Solution set $=\left\{\frac{5}{4}\right\}$

viii) $\frac{2x}{x-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{x-1}, \quad x\neq 1$

Multiplying both sides by $6(x-1)$

$$6\cancel{(x-1)}\times\frac{2x}{\cancel{x-1}}+{}^{2}\cancel{6}(x-1)\times\frac{1}{\cancel{3}}$$

$$=\cancel{6}(x-1)\times\frac{5}{\cancel{6}}+6\cancel{(x-1)}\times\frac{2}{\cancel{x-1}}$$

$$12x+2x-2=5x-5+12$$

$$12x+2x-5x=2-5+12$$

$$9x=9$$

$$x=\frac{9}{9}$$

$$\boxed{x=1}$$

Check:

Substituting $x=1$ in the given equator

$$\frac{2(1)}{1-1}+\frac{1}{3}=\frac{5}{6}+\frac{2}{1-1}$$

$$\frac{2}{0}+\frac{1}{3}=\frac{5}{6}+\frac{2}{0}$$

As $\frac{2}{0}$ is undefined, so $x=1$ cannot be a solution thus the given equation has no solution.

ix) $\frac{2}{x^2-1}-\frac{1}{x+1}=\frac{1}{x+1} \quad , \quad x\neq\pm 1$

$$\frac{2}{(x+1)(x-1)}-\frac{1}{x+1}=\frac{1}{x+1}$$

Multiplying both sides by $(x+1)(x-1)$

$$\cancel{(x+1)(x-1)}\times\frac{2}{\cancel{(x+1)(x-1)}}$$

$$-\cancel{(x+1)}(x-1)\times\frac{1}{\cancel{x+1}}=\frac{1}{\cancel{x+1}}\times\cancel{(x+1)}(x-1)$$

$$2-x+1=x-1$$

$$2+1+1=x+x$$

$$2x=4$$

$$x=\frac{4}{2}$$

$$x=2$$

Check:

Substituting $x=2$ in the given equation

$$\frac{2}{(2)^2-1}-\frac{1}{2+1}=\frac{1}{2+1}$$

$$\frac{2}{4-1}-\frac{1}{3}=\frac{1}{3}$$

$$\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$$

$\frac{1}{3}=\frac{1}{3}$ which is true, so

Solution Set $=\{2\}$

x) $\frac{2}{3x+6}=\frac{1}{6}-\frac{1}{2x+4} \quad , \quad x\neq -2$

$$\frac{2}{3(x+2)}=\frac{1}{6}-\frac{1}{2(x+2)}$$

Multiplying both sides by $6(x+2)$

$${}^{2}\cancel{6}\cancel{(x+2)}\times\frac{2}{\cancel{3}\cancel{(x+2)}}=$$

$$\frac{1}{\cancel{6}}\times\cancel{6}(x+2)-\frac{1}{\cancel{2}\cancel{(x+2)}}\times{}^{3}\cancel{6}\cancel{(x+2)}$$

$$4=x+2-3$$

$$4 = x - 1$$
$$4 + 1 = x$$
$$\boxed{x = 5}$$

Check:

Substituting $x = 5$ in the given equation

$$\frac{2}{3(5)+6} = \frac{1}{6} - \frac{1}{2(5)+4}$$
$$\frac{2}{15+6} = \frac{1}{6} - \frac{1}{10+4}$$
$$\frac{2}{21} = \frac{1}{6} - \frac{1}{14}$$
$$\frac{2}{21} = \frac{7-3}{42}$$
$$\frac{2}{21} = \frac{4}{42}$$
$$\frac{2}{21} = \frac{2}{21} \quad \text{which is true, so}$$

Solution Set $= \{5\}$

Q2. Solve each question and check for extraneous solution, if any.

i) $\sqrt{3x+4} = 2$

Squaring both sides

$$\left(\sqrt{3x+4}\right)^2 = (2)^2$$
$$3x + 4 = 4$$
$$3x = 4 - 4$$
$$3x = 0$$
$$x = \frac{0}{3}$$
$$\boxed{x = 0}$$

Check:

Substituting $x = 0$ in the given equation

$$\sqrt{3x+4} = 2$$
$$\sqrt{3(0)+4} = 2$$
$$\sqrt{0+4} = 2$$
$$\sqrt{4} = 2$$
$$2 = 2 \quad \text{which is true, so}$$

Solution Set $= \{0\}$

ii) $\sqrt[3]{2x-4} - 2 = 0$

$$\sqrt[3]{2x-4} = 2$$

Taking cube of both sides

$$\left(\sqrt[3]{2x-4}\right)^3 = (2)^3$$
$$2x - 4 = 8$$
$$2x = 8 + 4$$
$$2x = 12$$
$$x = \frac{\cancel{12}}{\cancel{2}}$$
$$\boxed{x = 6}$$

Check:

Putting $x = 6$ in the given equation.

$$\sqrt[3]{2x-4} - 2 = 0$$
$$\sqrt[3]{2(6)-4} - 2 = 0$$
$$\sqrt[3]{12-4} - 2 = 0$$
$$\sqrt[3]{8} - 2 = 0$$
$$\sqrt[3]{2^3} - 2 = 0$$
$$2 - 2 = 0$$
$$0 = 0 \quad \text{which is true, so}$$

Solution Set $= \{6\}$

iii) $\sqrt{x-3} - 7 = 0$

or $\sqrt{x-3} = 7$

Squaring both sides

$$\left(\sqrt{x-3}\right)^2 = (7)^2$$
$$x - 3 = 49$$
$$x = 49 + 3$$
$$x = 52$$

Check:

Putting $x = 52$ in the given equation

$$\sqrt{x-3} - 7 = 0$$

$$\sqrt{52-3} - 7 = 0$$

$$\sqrt{49} - 7 = 0$$

$$7 - 7 = 0$$

$0 = 0$ which true, so

Solution Set $= \{52\}$

iv) $2\sqrt{t+4} = 5$

$$\sqrt{t+4} = \frac{5}{2}$$

Squaring both sides

$$\left(\sqrt{t+4}\right)^2 = \left(\frac{5}{2}\right)^2$$

$$t + 4 = \frac{25}{4}$$

$$t = \frac{25}{4} - 4$$

$$= \frac{25-16}{4}$$

$$\boxed{t = \frac{9}{4}}$$

Check:

Putting $t = \frac{9}{4}$ in the given equation.

$$2\sqrt{t+4} = 5$$

$$2\sqrt{\frac{9}{4} + 4} = 5$$

$$2\sqrt{\frac{9+16}{4}} = 5$$

$$2\sqrt{\frac{25}{4}} = 5$$

$$\cancel{2}\left(\frac{5}{\cancel{2}}\right) = 5$$

$5 = 5$ which is true, so

Solution Set $= \left\{\frac{9}{4}\right\}$

v) $\sqrt[3]{2x+3} = \sqrt[3]{x-2}$

Taking cube of both sides

$$\left(\sqrt[3]{2x+3}\right)^3 = \left(\sqrt[3]{x-2}\right)^3$$

$$2x + 3 = x - 2$$

$$2x - x = -2 - 3$$

$$\boxed{x = -5}$$

Check:

Putting $x = -5$ in the given equation.

$$\sqrt[3]{2x+3} = \sqrt[3]{x-2}$$

$$\sqrt[3]{2(-5)+3} = \sqrt[3]{-5-2}$$

$$\sqrt[3]{-10+3} = \sqrt[3]{-7}$$

$\sqrt[3]{-7} = \sqrt[3]{-7}$ which is true, so

Solution Set $= \{-5\}$

vi) $\sqrt[3]{2-t} = \sqrt[3]{2t-28}$

Taking cube of both sides

$$\left(\sqrt[3]{2-t}\right)^3 = \left(\sqrt[3]{2t-28}\right)^3$$

$$2 - t = 2t - 28$$

$$2 + 28 = 2t + t$$

$$3t = 30$$

$$t = \frac{\cancel{30}}{\cancel{3}}$$

$$\boxed{t = 10}$$

Check:

Putting $t = 3$ in the given equation

$$\sqrt[3]{2-t} = \sqrt[3]{2t-28}$$

$$\sqrt[3]{2-10} = \sqrt[3]{2\times 10-28}$$

$$\sqrt[3]{-8}=\sqrt[3]{20-28}$$

$$\sqrt[3]{-8}=\sqrt[3]{-8} \text{ which is true, so}$$

Solution Set = $\{10\}$

vii) $\sqrt{2t+6}-\sqrt{2t-5}=0$ or

$$\sqrt{2t+6}=\sqrt{2t-5}$$

Squaring both sides

$$\left(\sqrt{2t+6}\right)^2=\left(\sqrt{2t-5}\right)^2$$

$$2t+6=2t-5$$

$$\cancel{2t}-\cancel{2t}+6=-5$$

$6=-5$ which is not possible, so

Solution Set = $\{\ \}$

viii) $\sqrt{\dfrac{x+1}{2x+5}}=2, \quad x\neq-\dfrac{5}{2}$

Squaring both sides

$$\left(\sqrt{\frac{x+1}{2x+5}}\right)^2=(2)^2$$

$$\frac{x+1}{2x+5}=4$$

$$x+1=4(2x+5)$$

$$x+1=8x+20$$

$$1-20=8x-x$$

$$-19=7x$$

$$\Rightarrow \quad \boxed{x=-\frac{19}{7}}$$

Check:

Putting $x=-\dfrac{19}{7}$ in the given equation

$$\sqrt{\frac{x+1}{2x+5}}=2$$

$$\sqrt{\frac{\frac{-19}{7}+1}{2\left(-\frac{19}{7}\right)+5}}=2$$

$$\sqrt{\frac{\frac{-19+7}{\cancel{7}}}{\frac{-38+35}{\cancel{7}}}}=2$$

$$\sqrt{\frac{\cancel{-12}}{\cancel{-3}}}=2$$

$$\sqrt{4}=2$$

$2=2$ which is true, so

Solution Set = $\left\{\dfrac{-19}{7}\right\}$

Definition

The absolute value of a real number 'a' denoted by $|a|$, is defined as

$$|a| = \begin{cases} a, & \text{if } a\geq 0 \\ -a, & \text{if } a<0 \end{cases}$$

e.g., $|6| = 6$, $|0| = 0$ and $|-6| = -(-6) = 6$

Some properties of Absolute Value

If $a, b \in R$, then

(i) $|a| \geq 0$

(ii) $|-a| = |a|$

(iii) $|ab| = |a|.|b|$

(iv) $\left|\dfrac{a}{b}\right|=\dfrac{|a|}{|b|}, \ b\neq 0$

Example

Solve and check, $|2x + 3| = 11$

Solution

By definition, depending on whether $(2x + 3)$ is positive or negative the given equation is equivalent to

$+(2x+3) = 11$ *or* $-(2x+3) = 11$

In practice, these two equations are usually written as

$2x+3 = +11$ or $2x+3 = -11$

$2x = 8$ or $2x = -14$

$x = 4$ or $x = -7$

Check

Substituting $x = 4$, in the original equation, we get

$|2(4)+3| = 11$

i.e., $11 = 11$, true

Now substituting $x = -7$, we have

$|2(-7)+3| = 11$

$|-11| = 11$

$11 = 11$, true

Hence $x = 4, -7$ are the solutions to the given equation.

Or Solution set = $\{-7, 4\}$

Example

Solve $|8x-3| = |4x+5|$

Solution

Since two numbers having the same absolute value are either equal or differ in sign, therefore, the given equation is equivalent to

$8x-3 = 4x+5$ or $8x-3 = -(4x+5)$

$8x-3 = 4x+5$ or $8x-3 = -4x-5$

$8x-4x = 5+3$ or $8x+4x = -5+3$

$4x = 8$ or $12x = -2$

$x = 2$ or $x = -1/6$

On checking we find that $x=2, x=\dfrac{-1}{6}$ both satisfy the original equation.

Hence the solution set $\left\{-\dfrac{1}{6}, 2\right\}$.

Sometimes it may happen that the solution(s) obtained do not satisfy the original equation. Such solution(s) (called extraneous) must be rejected. Therefore, it is always advisable to check the solutions in the original equation.

Example 3

Solve and check $|3x+10| = 5x+6$

Solution

The given equation is equivalent to

$\pm(3x+10) = 5x+6$

i.e., $3x+10 = 5x+6$ *or* $3x+10 = -(5x+6)$

$3x+10 = 5x+6$ *or* $3x+10 = -5x-6$

$3x-5x = 6-10$ *or* $3x+5x = -6-10$

$-2x = -4$ *or* $8x = -16$

$x = 2$ or x -2

On checking in the original equation we see that $x = -2$ does not satisfy it.

Hence the only solution is $x = 2$.

Exercise 7.2

Q1. Identify the following statements as True or False.

i) $|x| = 0$ has only one solution. (True)

ii) All absolute value equations have two solutions. (False)

iii) The equation $|x| = 2$ is equivalent to $x = 2$ or $x = -2$. (True)

iv) The equation $|x-4|=-4$ has no solution. (True)

v) The equation $|2x-3|=5$ is equivalent to $2x-3=5$ or $2x+3=5$ (False.)

Q2. Solve for 'x'.

i) $|3x-5|=4$

$\Rightarrow \quad +(3x-5)=4$ or $-(3x-5)=4$

$3x-5=4$ or $3x-5=-4$

$3x=4+5$ or $3x=-4+5$

$3x=9$ or $3x=1$

$x=3$ or $x=\frac{1}{3}$

<u>Check:</u>

Substituting $x=3$ in given equation

$|3(3)-5|=4$

$|9-5|=4$

$|4|=4$

$4=4$ which is true

Putting $x=\frac{1}{3}$ in given equation

$\left|3\left(\frac{1}{3}\right)-5\right|=4$

$|1-5|=4$

$|-4|=4$

$4=4$ which is true, so

Solution Set = $\left\{3,\frac{1}{3}\right\}$

ii) $\frac{1}{2}|3x+2|-4=11$

$\frac{1}{2}|3x+2|=11+4$

$\frac{1}{2}|3x+2|=15$

$|3x+2|=15\times 2$

$|3x+2|=30$

$+(3x+2)=30$ or $-(3x+2)=30$

$3x+2=30$ or $3x+2=-30$

$3x=30-2$ or $3x=-30-2$

$3x=28$ or $3x=-32$

$x=\frac{28}{3}$ or $x=\frac{-32}{3}$

<u>Check:</u>

Putting $x=\frac{28}{3}$ in the given equation

$\frac{1}{2}\left|\cancel{3}\left(\frac{28}{\cancel{3}}\right)+2\right|-4=11$

$\frac{1}{2}|28+2|-4=11$

$\frac{1}{2}|30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$ which is true

Now putting $x=-\frac{32}{3}$ in the given equation.

$\frac{1}{2}\left|\cancel{3}\left(-\frac{32}{\cancel{3}}\right)+2\right|-4=11$

$\frac{1}{2}|-32+2|-4=11$

$\frac{1}{2}|-30|-4=11$

$\frac{1}{2}(30)-4=11$

$15-4=11$

$11=11$ which is true, so

Hence Solution Set $=\left\{\frac{28}{3}, -\frac{32}{3}\right\}$

iii) $|2x+5|=11$

$+(2x+5)=11$ or $-(2x+5)=11$

$2x+5=11$ or $2x+5=-11$

$2x=11-5$ or $2x=-11-5$

$2x=6$ or $2x=-16$

$x=\frac{6}{2}$ or $x=\frac{-16}{2}$

$x=3$ or $x=-8$

Check:

Putting $x=3$ in the given equation.

$|2(3)+5|=11$

$|6+5|=11$

$|11|=11$

$11=11$ which is true

Now putting $x=-8$ in the given equation.

$|2(-8)+5|=11$

$|-16+5|=11$

$|-11|=11$

$11=11$ which is true, so

Solution Set $=\{3, -8\}$

iv) $|3+2x|=|6x-7|$

$\frac{|3+2x|}{|6x-7|}=1$

$\left|\frac{3+2x}{6x-7}\right|=1$

$+\left(\frac{3+2x}{6x-7}\right)=1$ or $-\left(\frac{3+2x}{6x-7}\right)=1$

$\frac{3+2x}{6x-7}=1$ or $\frac{3+2x}{6x-7}=-1$

$3+2x=6x-7$ or $3+2x=-6x+7$

$3+7=6x-2x$ or $2x+6x=7-3$

$10=4x$ or $8x=4$

$\Rightarrow x=\frac{10}{4}$ or $x=\frac{4}{8}$

$x=\frac{5}{2}$ or $x=\frac{1}{2}$

Check:

Putting $x=\frac{5}{2}$ in the given equation

$\left|3+\cancel{2}\left(\frac{5}{\cancel{2}}\right)\right|=\left|{}^{3}\cancel{6}\left(\frac{5}{\cancel{2}}\right)-7\right|$

$|3+5|=|15-7|$

$|8|=|8|$

$8=8$ which is true

Now putting $x=\frac{1}{2}$ in the given equation

$\left|3+\cancel{2}\left(\frac{1}{\cancel{2}}\right)\right|=\left|\cancel{6}\left(\frac{1}{\cancel{2}}\right)-7\right|$

$|3+1|=|3-7|$

$|4|=|-4|$

$4=4$ which is true, so

Solution Set $=\left\{\frac{5}{2},\frac{1}{2}\right\}$

v) $|x+2|-3=5-|x-2|$

$|x+2|+|x+2|=5+3$

$2|x+2|=8$

$|x+2|=\frac{\cancel{8}}{\cancel{2}}$

$|x+2|=4$

$+(x+2)=4$ or $-(x+2)=4$

$x+2=4$ or $x+2=-4$

$x=4-2$ or $x=-4-2$

$x=2$ or $x=-6$

Check:

Putting $x=2$ in the give equation

$|2+2|-3=5-|2+2|$

$|4|-3=5-|4|$

$4-3=5-4$

$1=1$ which is true

Now putting $x=-6$ in the given equation.

$|-6+2|-3=5-|-6+2|$

$|-4|-3=5-|-4|$

$4-3=5-4$

$1=1$ which is true, so

Solution Set = $\{2,-6\}$

vi) $\frac{1}{2}|x+3|+21=9$

$\frac{1}{2}|x+3|=9-21$

$\frac{1}{2}|x+3|=-12$

$|x+3|=-24$

As the value of absolute cannot be negative, so Solution Set = $\{\ \}$

vii) $\left|\frac{3-5x}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{2}{3}+\frac{1}{3}$

$\left|\frac{3-5x}{4}\right|=\frac{\cancel{3}}{\cancel{3}}$

$\left|\frac{3-5x}{4}\right|=1$

$+\left(\frac{3-5x}{4}\right)=1$ or $-\left(\frac{3-5x}{4}\right)=1$

$\frac{3-5x}{4}=1$ or $\frac{3-5x}{4}=-1$

$3-5x=4$ or $3-5x=-4$

$3-4=5x$ or $3+4=5x$

$-1=5x$ or $7=5x$

$x=-\frac{1}{5}$ or $x=\frac{7}{5}$

Check:

Putting $x=-\frac{1}{5}$ in the given equation

$\left|\frac{3-5\left(-\frac{1}{5}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{3+1}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$\left|\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$|1|-\frac{1}{3}=\frac{2}{3}$

$1-\frac{1}{3}=\frac{2}{3}$

$\frac{3-1}{3}=\frac{2}{3}$

$\frac{2}{3}=\frac{2}{3}$ which is true,

Now putting $x=\frac{7}{5}$ in the given equation

$\left|\frac{3-\cancel{5}\left(\frac{7}{\cancel{5}}\right)}{4}\right|-\frac{1}{3}=\frac{2}{3}$

$$\left|\frac{3-7}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$\left|-\frac{4}{4}\right|-\frac{1}{3}=\frac{2}{3}$$

$$|-1|-\frac{1}{3}=\frac{2}{3}$$

$$1-\frac{1}{3}=\frac{2}{3}$$

$$\frac{2}{3}=\frac{2}{3} \quad \text{which is true}$$

So, solution set = $\left\{-\frac{1}{5},\frac{7}{5}\right\}$

viii) $\left|\frac{x+5}{2-x}\right|=6$

$$+\left(\frac{x+5}{2-x}\right)=6 \quad \text{or} \quad -\left(\frac{x+5}{2-x}\right)=6$$

$$\frac{x+5}{2-x}=6 \quad \text{or} \quad \frac{x+5}{2-x}=-6$$

$$x+5=12-6x \quad \text{or} \quad x+5=12+6x$$

$$x+6x=12-5 \quad \text{or} \quad 5+12=6x-x$$

$$7x=7 \quad \text{or} \quad 17=5x$$

$$x=1 \quad \text{or} \quad x=\frac{17}{5}$$

Check:

Putting $x=1$ in the given equation.

$$\left|\frac{1+5}{2-1}\right|=6$$

$$\left|\frac{6}{1}\right|=6$$

$$|6|=6$$

$$6=6$$

Now putting $x=\frac{17}{5}$ in the given equation

$$\left|\frac{\frac{17}{5}+5}{2-\frac{17}{5}}\right|=6$$

$$\left|\frac{\frac{17+25}{\cancel{5}}}{\frac{10-17}{\cancel{5}}}\right|=6$$

$$\left|\frac{42}{-7}\right|=6$$

$$|-6|=6$$

$$6=6 \quad \text{which is true}$$

So, solution set = $\left\{1,\frac{17}{5}\right\}$

Definition of inequality

Let a, b be real numbers, then a is greater than b if the difference a – b is positive and we denote this order relation by the inequality a > b. An equivalent statement is that b is less than a, symbolized by b < a. Similarly, if a – b is negative, then a is less than b and expressed in symbols as a < b.

Properties of Inequalities

1. Law of Trichotomy

For any $a,b\in R$, one and only on of the following statements is true.

a < b or a = b, or a > b

An important special case of thi property is the case for $b = 0$, namely,

$a<0$ or $a=0$ or $a>0$ for any $a \in R$

2. Transitive Property

Let $a, b, c \in$ R.

(i) If $a > b$ and $b >$ c, then $a >$ c

(ii) If $a < b$ and $b < c$, then $a <$ c

3. Additive Closure Property

For $a, b, c \in$ R,

(i) If $a > b$, then $a + c > b + c$

If $a < b$, then $a + c > b + c$

(ii) If $a > 0$ and $b > 0$, then $a + b > 0$

If $a < 0$ and $b < 0$, then $a + b < 0$

4. Multiplicative Property

Let $a, b, c, d \in R$,

(i) If $a > 0$ and $b > 0$, then $ab > 0$, whereas $a < 0$ and $b < 0 \Rightarrow ab > 0$

(ii) If $a > b$ and $c > 0$, then $ac > bc$

Or if $a < b$ and $c > 0$, then $ac < bc$

(iii) If $a > b$ and $c < 0$, then $ac < bc$

Or if $a < b$ and $c < 0$, then $ac > bc$

The above property (iii) states that the sign of inequality is reversed if each side is multiplied by a negative real number.

(iv) If $a > b$ and $c > d$, then $ac > bd$

Example

Solve $9-7x>19-2x$, *where* $x\in R$.

Solution

$$9-7x>19-2x$$
$$9-5x>19$$
$$-5x>10$$
$$x<-2$$

Hence the solution set $= \{x \mid x<-2\}$

Example

Solve $\frac{1}{2}x-\frac{2}{3}\le x+\frac{1}{3}$, where $x\in R$.

Solution

$$\frac{1}{2}x-\frac{2}{3}\le x+\frac{1}{3}$$

To clear fractions we multiply each side by 6, the L.C.M of 2 and 3 and get

$$6\left[\frac{1}{2}x-\frac{2}{3}\right]\le 6\left[x+\frac{1}{3}\right]$$

$$6\times\frac{1}{2}x-\frac{6\times 2}{3}\le 6x+6\times\frac{1}{3}$$

or $\quad 3x-4\le 6x+2$

or $\quad -4-2\le 6x-3x$

or $\quad -6\le 3x$

or $\quad -\frac{6}{3}\le x$

$$-2\le x \Rightarrow x\ge -2$$

Hence the solution set $= \{x \mid x\ge -2\}$

Example

Solve the double inequality $-2<\frac{1-2x}{3}<1$, where $x\in R$.

Solution

The given inequality is a double inequality and represents two separate inequalities

$$-2<\frac{1-2x}{3} \quad \text{and} \quad \frac{1-2x}{3}<1$$

$$-2<\frac{1-2x}{3}<1$$

or $\quad -6<1-2x<3$

or $\quad -7<-2x<2$

or $\quad \frac{7}{2}>x>-1$

i.e., $-1<x<3.5$

Hence S.S $= \{x \mid -1<x<3.5\}$

Example

Solve the inequality $4x-1\le 3\le 7+2x$, where $x\in R$.

Solution

The given inequality holds if and only if both the separate inequalities $4x-1\le 3$ and $3\le 7+2x$ hold. We solve each of these inequalities separately.

The first inequality $4x-1\le 3$ gives

$4x\le 4$ i.e., $x\le 1$ ....(i)

$3 \le 7 + 2x \Rightarrow -4 \le 2x$

i.e.

$-2 \le x \Rightarrow x \ge -2$ ....(ii)

Combining (i) and (ii) we have

$-2 \le x \le 1$

Thus the solution set = $\{x \mid -2 \le x \le 1\}$.

Exercise 7.3

Q1. Solve the following in equalities.

i) $3x+1<5x-4$

$1+4<5x-3x$

$5<2x$

$\frac{5}{2}<x$

or $x>\frac{5}{2}$

Solution Set = $\left\{x \mid x>\frac{5}{2}\right\}$

ii) $4x-10.3 \le 21x-1.8$

$4x-21x \le 10.3-1.8$

$-17x \le 8.5$

$17x \ge -8.5$

$x \ge -\frac{8.5}{17}$

$x \ge -0.5$

Solution Set = $\{x \mid x \ge -0.5\}$

iii) $4-\frac{1}{2}x \ge -7+\frac{1}{4}x$

$4+7 \ge \frac{1}{4}x+\frac{1}{2}x$

$11 \ge \frac{x+2x}{4}$

$11 \ge \frac{3}{4}x$

$\frac{11\times 4}{3} \ge x$

$\frac{44}{3} \ge x$

or $x \le \frac{44}{3}$

Solution Set = $\left\{x \mid x \le \frac{44}{3}\right\}$

iv) $x-2(5-2x) \ge 6x-3\frac{1}{2}$

$x-2(5-2x) \ge 6x-\frac{7}{2}$

Multiplying both sides by 2

$2x-4(5-2x) \ge 12x-7$

$2x-20+8x \ge 12x-7$

$2x+8x-12x \ge 20-7$

$-2x \ge 13$

$2x \le -13$

$x \le -\frac{13}{2}$

Solution Set = $\left\{x \mid x \le -\frac{13}{2}\right\}$

v) $\frac{3x+2}{9}-\frac{2x+1}{3}>-1$

Multiplying both sides by 9

$3x+2-3(2x+1)>-9$

$3x+2-6x-3>-9$

$-3x-1>-9$

$-3x>1-9$

$-3x>-8$

$x<\frac{-8}{-3}$

$x<\frac{8}{3}$

$$\text{Solution Set} = \left\{x \mid x < \frac{8}{3}\right\}$$

vi) $3(2x+1)-2(2x+5)<5(3x-2)$

$6x+3-4x-10<15x-10$

$2x-7<15x-10$

$10-7<15x-2x$

$3<13x$

$\frac{3}{13}<x$

or $x>\frac{3}{13}$

$$\text{Solution Set} = \left\{x \mid x > \frac{3}{13}\right\}$$

vii) $3(x-1)-(x-2)>-2(x+4)$

$3x-3-x+2>-2x-8$

$2x-1>-2x-8$

$2x+2x>1-8$

$4x>-7$

$x>-\frac{7}{4}$

$$\text{Solution Set} = \left\{x \mid x > -\frac{7}{4}\right\}$$

viii) $2\frac{2}{3}x+\frac{2}{3}(5x-4)>-\frac{1}{3}(8x+7)$

$\frac{8}{3}x+\frac{2}{3}(5x-4)>-\frac{1}{3}(8x+7)$

Multiplying both sides by 3

$8x+2(5x-4)>-(8x+7)$

$8x+10x-8>-8x-7$

$18x-8>-8x-7$

$18x+8x>8-7$

$26x>1$

$x>\frac{1}{26}$

$$\text{Solution Set} = \left\{x \mid x > \frac{1}{26}\right\}$$

Q2. Solve the following inequalities.

i) $-4<3x+5<8$

$-4<3x+5$ and $3x+5<8$

$-4-5<3x$ and $3x<8-5$

$-9<3x$ and $3x<3$

$-\frac{9}{3}<x$ and $x<\frac{3}{3}$

$-3<x$ and $x<1$

$-3<x<1$

$$\text{Solution Set} = \{x \mid -3<x<1\}$$

ii) $-5\le\frac{4-3x}{2}<1$

$-5\le\frac{4-3x}{2}$ and $\frac{4-3x}{2}<1$

$-10x\le 4-3x$ and $4-3x<2$

$-10-4\le-3x$ and $-3x<2-4$

$-14\le-3x$ and $-3x<-2$

$14\ge 3x$ and $3x>2$

$\frac{14}{3}\ge x$ and $x>\frac{2}{3}$

$\frac{14}{3}\ge x>\frac{2}{3}$

$$\text{Solution Set} = \left\{x \mid \frac{14}{3}\ge x>\frac{2}{3}\right\}$$

iii) $-6<\frac{x-2}{4}<6$

$-6<\frac{x-2}{4}$ and $\frac{x-2}{4}<6$

$-24<x-2$ and $x-2<24$

$-24+2<x$ and $x<24+2$

$-22<x$ and $x<26$

$$\text{Solution Set} = \{x \mid -22<x<26\}$$

iv) $3\ge\frac{7-x}{2}\ge 1$

$$3\geq\frac{7-x}{2} \quad \text{and} \quad \frac{7-x}{2}\geq 1$$
$$6\geq 7-x \quad \text{and} \quad 7-x\geq 2$$
$$6-7\geq -x \quad \text{and} \quad -x\geq 2-7$$
$$-1\geq -x \quad \text{and} \quad -x\geq -5$$
$$1\leq x \quad \text{and} \quad x\leq 5$$
$$1\leq x\leq 5$$

Solution Set $=\{x|1\leq x\leq 5\}$

v) $3x-10\leq 5<x+3$
$$3x-10\leq 5 \quad \text{and} \quad 5<x+3$$
$$-5-10\leq -3x \quad \text{and} \quad -x<3-5$$
$$-15\leq -3x \quad \text{and} \quad -x<-2$$
$$15\geq 3x \quad \text{and} \quad x>2$$
$$5\geq x \quad \text{and} \quad x>2$$
$$5\geq x>2$$

Solution Set $=\{x|5\geq x>2\}$

vi) $-3\leq\frac{x-4}{-5}<4$
$$-3\leq\frac{x-4}{-5} \quad \text{and} \quad \frac{x-4}{-5}<4$$
$$\Rightarrow \quad 3\geq\frac{x-4}{5} \quad \text{and} \quad \frac{x-4}{5}>-4$$
$$15\geq x-4 \quad \text{and} \quad x-4>20$$
$$15+4\geq x \quad \text{and} \quad x>4-20$$
$$19\geq x \quad \text{and} \quad x>-16$$
$$19\geq x\geq -16$$

Solution Set $=\{x|19\geq x>-16\}$

vii) $1-2x<5-x\leq 25-6x$
$$1-2x<5-x \quad \text{and} \quad 5-x\leq 25-6x$$
$$1-5\leq 2x-x \quad \text{and} \quad 6x-x\leq 25-5$$
$$-4<x \quad \text{and} \quad 5x\leq 20$$
$$-4<x \quad \text{and} \quad x\leq 4$$
$$-4<x\leq 4$$

Solution Set $=\{x|-4<x\leq 4\}$

viii) $3x-2<2x+1<4x+17$
$$3x-2<2x+1 \quad \text{and} \quad 2x+1<4x+17$$
$$-2-1<2x-3x \quad \text{and} \quad 2x-4x<17-1$$
$$-3<-x \quad \text{and} \quad -2x<16$$
$$3>x \quad \text{and} \quad 2x>-16$$
$$3>x \quad \text{and} \quad x>-8 \quad 3>x>-8$$

Solution Set $=\{x|3>x>-8\}$

Review Exercise 7

Q3. Answer the following short questions.

i) Define a linear inequality in one variable.

Ans. **Linear Inequality in one variable**

Let a, b be real numbers, then a is greater than b if the difference a – b is positive and we denote this order relation by the inequality a > b. An equivalent statement is that b is less than a, symbolized by b < a. Similarly, if a – b is negative, then a is less than b and expressed in symbols as $a<b$.

ii) State the trichotomy and transitive properties of inequality.

Ans. **Trichotomy Property of inequality**

For any $a,b\in R$, one and only one of the following statements is true.

$a<b$ or $a=b$, or $a>b$

Transitive Property of inequality

Let $a,b,c\in R$

i) If $a>b$ and $b>c$, then $a>c$

ii) If $a>b$ and $b<c$, then $a<c$

iii) **The formula relating degrees Fahrenheit to degrees Celcius is $F=\frac{9}{5}C+32$. For what value of C is $F<0$.**

Ans. According to formula "F" will be zero, if $\frac{9}{5}C+32=0$

$\frac{9}{5}C=-32$

$C=-\frac{32}{9}\times5$

$C=-\frac{160}{9}$

To get $F<0$ i.e. negative $C<-\frac{160}{9}$

v) **Seven times the sum of an integer and 12 is at least 50 and at most 60. Write and solve the inequality that expresses this relationship.**

Ans. Let the required integer be x then

$50\le x+12\le 60$

$50\le x+12$ and $x+12\le 60$

$50-12\le x$ and $x\le 60-12$

$38\le x$ and $x\le 48$

$38\le x\le 48$

Q4. **Solve each of the following and check for extraneous solution, if any.**

i) $\sqrt{2t+4}=\sqrt{t-1}$

Squaring both sides

$\left(\sqrt{2t+4}\right)^2=\left(\sqrt{t-1}\right)^2$

$2t+4=t-1$

$2t-t=-1-4$

$t=-5$

Check:

$\sqrt{2t+4}=\sqrt{t-1}$

$\sqrt{2(-5)+4}=\sqrt{-5-1}$

$\sqrt{-10+4}=\sqrt{-6}$

$\sqrt{-6}=\sqrt{-6}$ Which is true, so solution Set = $\{-5\}$

ii) $\sqrt{3x-1}-2\sqrt{8-2x}=0$

$\sqrt{3x-1}=2\sqrt{8-2x}$

Squaring both sides

$\left(\sqrt{3x-1}\right)^2=\left(2\sqrt{8-2x}\right)^2$

$3x-1=4(8-2x)$

$3x-1=32-8x$

$3x+8x=32+1$

$11x=33$

$x=\frac{\cancel{33}}{\cancel{11}}$

$x=3$

Check:

$\sqrt{3x-1}-2\sqrt{8-2x}=0$

$\sqrt{3(3)-1}-2\sqrt{8-2(3)}=0$

$\sqrt{9-1}-2\sqrt{8-6}=0$

$\sqrt{8}-2\sqrt{2}=0$

$2\sqrt{2}-2\sqrt{2}=0$

$0=0$ Which is true, so

solution set = $\{3\}$

Q5. Solve for x

i) $|3x+14|-2=5x$

$|3x+14|=5x+2$

$\pm(3x+14)=5x+2$

$3x+14=\pm(5x+2)$

$3x+14=5x+2$ or $3x+14=-5x-2$

$3x-5x=2-14$ or $3x+5x=-2-14$

$-2x=-12$ or $8x=-16$

$x=\frac{12}{2}$ or $x=-\frac{16}{8}$

$x=6$ or $x=-2$

Check:

Put $x=6$ in

$$|3x+14|-2=5x$$
$$|3(6)+14|-2=5(6)$$
$$|18+14|-2=30$$
$$|32|-2=30$$
$$30-2=30$$
$30=30$, which is true

Now put $x=-2$

$$|3(-2)+14|-2\neq 5(-2)$$
$$|-6+14|-2\neq -10$$
$$|8|-2\neq -10$$
$$8-2\neq -10$$
$6\neq -10$ which is not true

So, Solution Set $=\{6\}$

ii) $$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$
$$\frac{|x-3|}{|x+2|}=\frac{3}{2}$$
$$\left|\frac{x-3}{x+2}\right|=\frac{3}{2}$$
$$\pm\left(\frac{x-3}{x+2}\right)=\frac{3}{2}$$

or $$\frac{x-3}{x+2}=\pm\frac{3}{2}$$
$$\frac{x-3}{x+2}=\frac{3}{2} \quad \text{or} \quad \frac{x-3}{x+2}=-\frac{3}{2}$$
$$2(x-3)=3(x+2) \text{ or } 2(x-3)=-3(x+2)$$
$$2x-6=3x+6 \text{ or } 2x+3x=6-6$$
$$-x=12 \text{ or } 5x=0$$
$$x=-12 \text{ or } x=0$$

Check:

Put $x=-12$

$$\frac{1}{3}|x-3|=\frac{1}{2}|x+2|$$
$$\frac{1}{3}|-12-3|=\frac{1}{2}|-12+2|$$
$$\frac{1}{3}|-15|=\frac{1}{2}|-10|$$
$$\frac{15}{3}=\frac{10}{2}$$
$5=5$, which is true

Now put $x=0$

$$\frac{1}{3}|0-3|=\frac{1}{2}|0+2|$$
$$\frac{1}{3}|-3|=\frac{1}{2}|2|$$
$$\frac{3}{3}=\frac{2}{2}$$
$1=1$, which is true

So, Solution Set $=\{-12,0\}$

Q6. Solve the following inequality.

i) $$-\frac{1}{3}x+5\leq 1$$
$$-\frac{1}{3}x\leq 1-5$$
$$-\frac{1}{3}x\leq -4$$

Multiplying both sides by -3

$$x\geq 12$$

Solution Set $=\{x/x\geq 12\}$

ii) $$-3<\frac{1-2x}{5}<1$$
$$-3<\frac{1-2x}{5} \quad \text{and} \quad \frac{1-2x}{5}<1$$

Multiplying both sides by 5

$$-15<1-2x \quad \text{and} \quad 1-2x<5$$
$$-15-1<-2x \quad \text{and} \quad -2x<5-1$$
$$-16<-2x \quad \text{and} \quad -2x<4$$

Multiplying both sides by -1

$$16>2x \quad \text{and} \quad 2x>-4$$
$$\frac{16}{2}>x \quad \text{and} \quad x>\frac{-4}{2}$$
$$8>x \quad \text{and} \quad x>-2$$
$$8>x>-2$$

Solution Set $=\{x/8>x>-2\}$

Objective

1. *Which of the following is the solution of the inequality* $3-4x \le 11$?
   (a) $x \ge -8$
   (b) $x \ge -2$
   (c) $x \ge \frac{-14}{4}$
   (d) None of these

2. A statement involving any of the symbols <, > or ≤ or ≥ is called:
   (a) Equation (b) Identity
   (c) Inequality (d) Linear equation

3. $x =$ ________ is a solution of the inequality $-2 < x < \frac{3}{2}$
   (a) -5 (b) 3
   (c) 0 (d) $\frac{5}{2}$

4. If x is not larger than 10, then
   (a) $x \ge 8$ (b) $x \le 10$
   (c) $x < 10$ (d) $x > 10$

5. If the capacity c of an elevator is at most 1600 pounds, then __
   (a) $c < 1600$ (b) $c \ge 1600$
   (c) $c \le 1600$ (d) $c > 1600$

6. $x = 0$ is a solution of the inequality
   (a) $x > 0$ (b) $3x + 5 < 0$
   (c) $x + 2 < 0$ (d) $x - 2 < 0$

7. The linear equation in one variable x is:
   (a) $ax + b = 0$
   (b) $ax^2 + bx + c = 0$
   (c) $ax + by + c = 0$
   (d) $ax^2 + by^2 + c = 0$

8. *An inconsistent equation is that* whose solution set is:
   (a) Empty (b) Not empty
   (c) Zero (d) None of these

9. Absolute value of a real number a is defined as:
   (a) $|a| = \begin{cases} a & \text{if } a \ge 0 \\ -a, & \text{if } a < 0 \end{cases}$
   (b) $|a| = \begin{cases} a & \text{if } a \le 0 \\ -a & \text{if } a > 0 \end{cases}$
   (c) $|a| = \begin{cases} a & \text{if } a > 0 \\ -a & \text{if } a < 0 \end{cases}$
   (d) None of these

10. $|x| = a$ is equivalent to:
    (a) $x = a$ or $x = -a$
    (b) $x = \frac{1}{a}$ or $x = \frac{-1}{a}$
    (c) $x = a$ or $x = \frac{-1}{a}$
    (d) None of these

11. A linear inequality in one variable x is:
    (a) $ax + b > 0, \quad a \ne 0$
    (b) $ax^2 + bx + c < 0, a \ne 0$
    (c) $ax + by + c > 0, \quad a \ne 0$
    (d) $ax^2 + by^2 + c < 0, a \ne 0$

12. Law of Trichotomy is … $(a, b \in R)$
    (a) $a < b$ or $a = b$ or $a > b$
    (b) $a < b$ or $a = b$
    (c) $a < b$ or $a > b$
    (d) *None of these*

13. Transitive law is____

(a) $a < b$ and $b < c$, then $a < c$

(b) $a > b$ and $b < c$, then $a > c$

(c) $a > b$ and $b < c$, then $a > c$

(d) None of these

14. If $a > b$, $c > 0$ then:

(a) $ac < bc$ (b) $ac > bc$

(c) $ac = bc$ (d) None

15. If $a > b$, $c > 0$ then:

(a) $\frac{a}{c} > \frac{b}{c}$ (b) $\frac{a}{c} < \frac{b}{c}$

(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{b}{c} \neq \frac{b}{c}$

16. If $a > b$, $c < 0$, then:

(a) $\frac{a}{c} < \frac{b}{c}$ (b) $\frac{a}{c} > \frac{b}{c}$

(c) $\frac{a}{c} = \frac{b}{c}$ (d) $\frac{a}{c} \leq \frac{b}{c}$

17. If $a, b \in R$ then:

(a) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (b) $|ab| = \frac{|a|}{|b|}$

(c) $\left|\frac{b}{a}\right| = \frac{|b|}{|a|}$ (d) None of these

18. When the variable in an equation occurs under a radical, the equation is called a ___equation.

(a) Radical (b) Absolute value

(c) Linear (d) None of these

19. $|x| = 0$ has only ___ solution.

(a) one (b) two

(c) three (d) none of these

20. The equation $|x| = 2$ is equivalent to

(a) $x = 2$ or $x = -2$

(b) $x = -2$ or $x = -2$

(c) $x = 2$ or $x = \frac{1}{2}$

(d) $x = 2$ or $x = \frac{-1}{2}$

21. An ___ is equation that is satisfied by every number for which both sides are defined:

(a) Identity (b) Conditional

(c) Inconsistent (c) None

22. An___ equation is an equation whose solution set is the empty set:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

23. A ___ equation is an equation that is satisfied by atleast one number but is not an identity:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

24. $x + 4 = 4 + x$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

25. $2x + 1 = 9$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

26. $x = x + 5$ is ___ equation:

(a) Identity (b) Conditional

(c) Inconsistent (d) None

27. Equations having exactly the same solution are called ___ equations.

(a) equivalent (b) Linear

(c) Inconsistent (c) None

28. A solution that does not satisfy the original equation is called ____ solution:

(a) Extraneous (b) Root

(c) General (d) None

ANSWER KEY

1.	b	2.	c	3.	c	4.	b	5.	c
6.	d	7.	a	8.	a	9.	a	10.	a
11.	a	12.	a	13.	a	14.	b	15.	a
16.	a	17.	a	18.	a	19.	a	20.	a
21.	a	22.	c	23.	b	24.	a	25.	b
26.	c	27.	a	28.	a				